Solving One-Variable First Degree Inequalities

- Solving inequalities is very similar to solving equations when the exponent on the variable is one.
- For example, the following equation is solved as:

$$3x + 4 = 22 \qquad x = \frac{18}{3}$$

$$3x = 22 - 4$$

$$3x = 18 \qquad x = 6$$

- In this case, there is only one correct answer, **x = 6**.

- If we replace the equals sign with an inequality like **<**, we get.

$$3x + 4 < 22 \qquad x < \frac{18}{3}$$

$$3x < 22 - 4$$

$$3x < 18 \qquad x < 6$$

- In this case, for **x < 6**, there are infinite number of correct answers, provided that the answer is less than 6.

Solving One-Variable First Degree Inequalities

• Things start to get interesting when we have to divide both sides of an inequality by a negative number to isolate the variable x.

• When this situation arises, we must reverse the direction of the inequality to accommodate the division by a negative. If you multiply both sides by a negative you will have to reverse the inequality symbol too.

• Let's try one of these.

Solving One-Variable First Degree Inequalities

- $-5(x-2) \geq 15$

$-5x + 10 \geq 15$

$-5x \geq 15 - 10$

$-5x \geq 5$

$x \leq \frac{5}{-5}$ **Reverse the direction of the inequality because we are dividing by -5**

$x \leq -1$

Here, x is less than or equal to -1

• Finally, sometimes you will have to solve for x when there are two inequalities. Let's go through one of those step by step.

Solving One-Variable First Degree Inequalities

- $13 \le 2x - 5 \le 21$

First, we will move the -5 out of the middle

$$13 + 5 \le 2x \le 21 + 5$$

$$18 \le 2x \le 26$$

Next, we will divide each term by 2.

$$\frac{18}{2} \le \frac{2x}{2} \le \frac{26}{2}$$

$$9 \le x \le 13$$

As an interval, this can be written as:

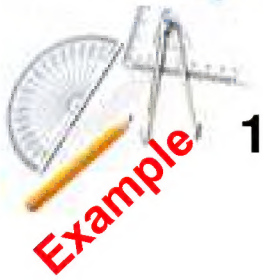
$$x \in [9,13]$$

This means that any number between and including 9 and 13 is a solution

Solving One-Variable First Degree Inequalities

- When you do the word problems in this section, refer to the vocabulary words you learned in the previous section on inequalities and the word problems you did in the solving equations section.

- We are ready to do some more challenging questions.



1. Solve the following inequality.

$$2x - 1 < 7$$

• **Step 1: Collect the terms with an "x" on the left, and the other terms on the right of the inequality symbol.**

$$2x - 1 < 7$$

$$2x < 7 + 1$$

$$2x < 8$$

• **Step 2: Isolate the "x" and solve**

$$2x < 8$$

$$x < \frac{8}{2}$$

$$x < 4 \quad \text{so x is less than 4}$$

2. Solve the following inequality.

$$-3x - 2 \leq 10$$

- **Step 1:** Collect the terms with an "x" on the left, and the other terms on the right of the inequality symbol.

$$-3x - 2 \leq 10$$

$$-3x \leq 10 + 2$$

$$-3x \leq 12$$

- **Step 2:** Isolate the "x" and solve. **Remember we have to divide by -3, so the direction of the inequality must be reversed!**

$$-3x \leq 12$$

$$x \geq \frac{12}{-3}$$

$$x \geq -4$$ so x is greater than or equal to -4

3. Solve the following inequality and give the solution set in interval notation.

$$5x - 12 > 3x + 2$$

- **Step 1: Collect the terms with an "x" on the left, and the other terms on the right of the inequality symbol.**

$$5x - 12 > 3x + 2$$
$$5x - 3x > 2 + 12$$
$$2x > 14$$

- **Step 2: Isolate the "x" and solve**

$$2x > 14$$
$$x > \frac{14}{2}$$
$$x > 7$$

- **Step 3: Express x > 7 in interval notation**

$$x > 7$$
$$x \in \left]7, \infty^{+}\right[$$ is the interval notation

4. Solve the following inequality and give the solution set in interval notation.

$$5 < 2x + 1 < 9$$

- **Step 1: Get the +1 out of the middle of the inequality**

$$5 < 2x + 1 < 9$$
$$5 - 1 < 2x < 9 - 1$$
$$4 < 2x < 8$$

- **Step 2: Isolate the "x" by dividing every part of the inequality by 2**

$$4 < 2x < 8$$
$$\frac{4}{2} < \frac{2x}{2} < \frac{8}{2}$$
$$2 < x < 4$$

- **Step 3: Express 2 < x < 4 in interval notation**

$$2 < x < 4$$
$$x \in \left]2, 4\right[$$ is the interval notation

5. In a group of at least 30 children, there are 4 more boys than girls. What is the minimum number of boys in this group of children?

- **Step 1: Set up the inequality**

x: # of girls

$x + 4$: # of boys

# of boys + # of girls $\geq$ 30 children

$$(x + 4) + x \geq 30$$

- **Step 2: Solve the inequality**

$$x + 4 + x \geq 30$$
$$2x \geq 30 - 4$$
$$2x \geq 26$$
$$x \geq \frac{26}{2}$$
$$x \geq 13$$

there must be at least 13 girls

- **Step 3: Calculate the minimum number of boys**

$$\text{\# of boys} = x + 4$$
$$= 13 + 4$$
$$= 17$$

$x \geq 17$ there is a minimum of 17 boys